

5 Sets. Union, Intersection, and Set Difference

32. Let $A = \{1, 2, 5, 6, 7, 10, 11, 12\}$, $B = \{x \mid x \text{ is an odd integer and } 3 \leq x \leq 10\}$, and $C = \{a, b, c, 4, 5, 6\}$.

(a) Fill in the blanks $_\$ with the most appropriate symbol ($\in, \notin, \subseteq, \not\subseteq$):
 $7 _\ A$, $6 _\ B$, $10 _\ B$, $4 _\ A$, $b _\ C$, $b _\ B$, $\{2, 5, 12\} _\ A$, $\{3, 5, 6\} _\ B$, $\{a, b, 5\} _\ C$.

(b) Calculate $A \cup B$, $A \setminus B$, $\mathcal{P}(A \cap B)$, $(A \cap B) \times \{a, b\}$.

33. Let $A = \{2n - 1 : n \in \mathbb{Z}\}$ be the set of all odd integers and let $B = \{(2n - 1)^3 : n \in \mathbb{Z}\}$ be the set of all cubes of odd integers. Show that $B \subseteq A$.

34. (i) Show that for any sets A , B , and C , the following holds:

$$A \subseteq C \wedge B \subseteq C \Rightarrow A \cup B \subseteq C.$$

(ii) Determine the following sets:

- $\{\emptyset, \{\emptyset\}\} \setminus \emptyset$.
- $\{\emptyset, \{\emptyset\}\} \setminus \{\emptyset\}$.
- $\{\emptyset, \{\emptyset\}\} \setminus \{\{\emptyset\}\}$.

35. Find all sets A, B, C such that

$$B \setminus A = A \cup C = C \cap B = \{1\}.$$

36. Is the following statement true for any sets A , B , and C :

$$\text{If } A \cap B \subseteq \overline{C} \text{ and } A \cup C \subseteq B, \text{ then } A \cap C = \emptyset.$$

37. Justify whether the following statement holds for any sets A , B , and C :

$$A \cap B = A \cap C \Rightarrow B = C.$$

38. Let A and B be arbitrary sets. Prove that:

$$(A \cap B) \cup (B \setminus A) = B.$$

39. Let X and Y be subsets of a universal set U . Show that

$$(X \cup Y)^c = X^c \cap Y^c.$$

40. (exam, November 2021.) (a) Let A and B be given sets. Show that $A \cap (B \setminus A) = \emptyset$. Explain each step of your proof.

(b) Let A and B be subsets of a universal set U . Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$. Explain each step of your proof.

41. Justify whether the following statement holds for any sets A , B , and C :

$$(A \cup B) \cap (A \cup C) \subseteq \overline{A} \cup C.$$

Explain each step of your proof.

42. Let A , B , and C be arbitrary sets. Prove that

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

43. Show that for any sets A , B , and C , the following holds:

$$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C).$$

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.